

# Letters to the Editor

The Board of Editors does not hold itself responsible for opinions expressed in the letters published in this section. The notes containing short reports of original investigations communicated to this section should not contain many figures and should not exceed 500 words in length. The contributions reaching the Secretary by the 15th of any month may be expected to appear in the issue for the next month. No proof will be sent to the author.

17

## MATRIX RELATIONS BETWEEN DIRECTION COSINES AND MILLERIAN INDICES

J. K. GHOSE

DEPARTMENT OF PHYSICS, ST JOHN'S COLLEGE, AGRA.

(Received May 15, 1965)

For magnetic measurements in triclinic crystals, directions are usually specified by the Millerian indices of one or two planes, while for many calculations, the direction cosines with reference to some orthogonal axes  $x, y, z$ , are necessary. As an example, suppose the  $z$  axis coincides with the crystallographic  $c$  axis and the  $x$  axis lies in the  $a-c$  plane close to the  $a$  axis. The cosines of the angles between the various axes are as follows.

	a	b	c
$x$	$\sin \beta$	$(\cos \gamma - \cos \alpha \cos \beta) / \sin \beta$	0
$y$	0	$M / \sin \beta$	0
$z$	$\cos \beta$	$\cos \alpha$	1

where  $M = (1 - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma + 2 \cos \alpha \cos \beta \cos \gamma)^{1/2}$  and is positive. The matrix  $[J]$  of transformation from  $x, y, z$  to  $a, b, c$ , is the matrix formed by the trigonometrical quantities as arranged in the above table (Ghose, 1964). These coordinates  $x, y, z$ , have a slight advantage compared to those discussed by U. S. Ghosh and Mitra (1965), because the determinant of  $[J]$  is positive. The values of  $[J]$  and  $[J]^{-1}$  need be calculated only once for a particular variety of crystal. It may be verified that

$$[J]^{-1} = \begin{bmatrix} \frac{1}{\sin \beta} & -\frac{\cos \alpha \cos \beta - \cos \gamma}{M \sin \beta} & 0 \\ 0 & \frac{\sin \beta}{M} & 0 \\ -\frac{\cos \beta}{\sin \beta} & \frac{\cos \beta \cos \gamma - \cos \alpha}{M \sin \beta} & 1 \end{bmatrix}$$

As shown by Ghose (1964), a unit vector  $[\xi \eta \zeta]$  in  $x, y, z$  coordinates, transforms to a unit covariant vector  $[V_p]$  and a unit contravariant vector  $[V^p]$  in  $a, b, c$  coordinates such that

$$[\xi \eta \zeta] = [V_a \ V_b \ V_c][J]^{-1} = [J][V^a V^b V^c]' \quad \dots (1)$$

It may be noted that  $V_a, V_b, V_c$  are cosines of the angles made by the unit vector with  $a, b, c$  axes while  $\xi, \eta, \zeta$  are those made with the orthogonal  $x, y, z$  axes.

If  $a:b:c$  be the axial length ratio and  $(hkl)$  be the Miller indices of a plane, then any contravariant vector lying in this plane, is given by

$$[U^a \ U^b \ U^c] = \left[ (m+n) \frac{a}{h} - \frac{mb}{k} - \frac{nc}{l} \right]$$

where  $m$  and  $n$  are any two numbers. For a covariant unit vector  $[V_p]$  normal to this plane,  $[V_p][U^p]$  is zero for all values of  $m$  and  $n$ . Hence we get

$$[V_a \ V_b \ V_c] = Q \begin{bmatrix} h & k & l \\ a & b & c \end{bmatrix} \quad \dots (2)$$

where  $Q$  is a suitable multiplier to make the magnitude of the vector equal to unity.

Again if two planes  $(hkl)$  and  $(h'k'l')$  are both kept perpendicular to the plane of measurement, then the normal to the latter lies in both  $(hkl)$  and  $(h'k'l')$  planes and is given by the contravariant vector

$$[V^a V^b V^c]' = Q_1 [a(kl' - k'l) \ b(lh' - l'h) \ c(hk' - h'k)]' \quad \dots (3)$$

where  $Q_1$  is a suitable multiplier.

Values of  $\xi, \eta, \zeta$  may be found by combining either eq. (2) or eq. (3) with eq.(1). If  $\xi_1, \eta_1, \zeta_1$  are the values obtained when  $Q$  or  $Q_1$  is arbitrarily taken as 1, then the actual value of  $Q$  or  $Q_1$  is  $(\xi_1^2 + \eta_1^2 + \zeta_1^2)^{-1/2}$ . When only the ratio of two direction cosines is required,  $Q$  need not be calculated.

This matrix method of calculating the direction cosines seems to be simpler and quicker than other methods.

#### REFERENCES

- Ghose, J. K., 1964, *Ind. Jour. pure appl. Phys.*, **2**, 94.  
 Ghosh, U. S. and Mitra, S., 1964, *Ind. J. Phys.*, **38**, 19.